CSCI 410: Modeling and Simulation

Written Assignment 1 Key

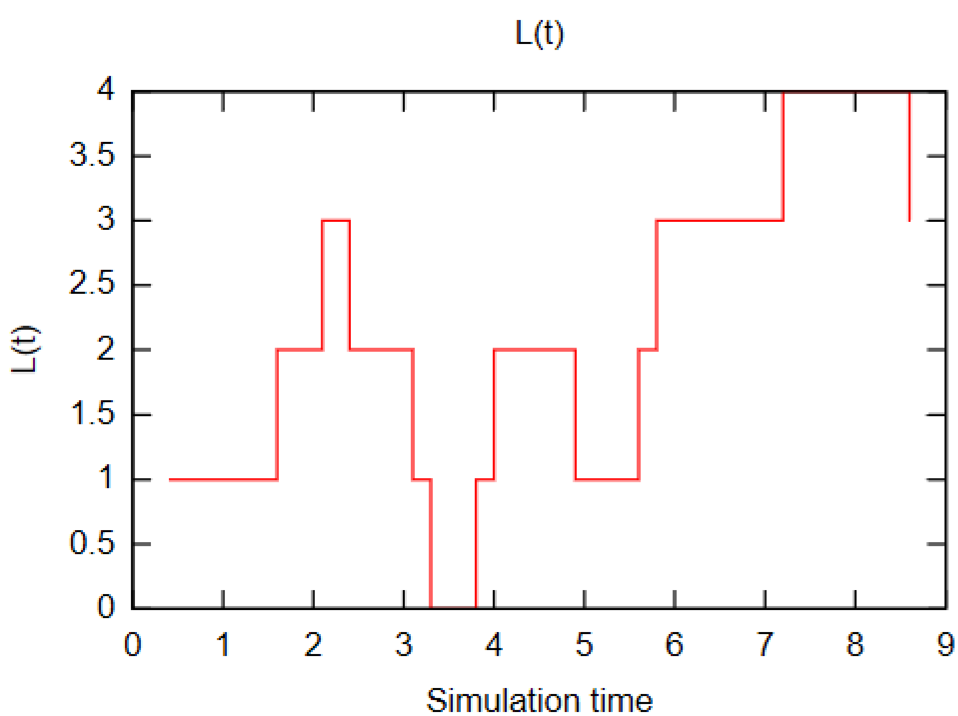
(103 pts in total)

1. For the single-server queueing system in Section 1.4, define L(t) to be the total number of customers in the system at time t (including the queue and the customer in service at time t, if any).
   (a) **(4 pts)** Is it true that L(t) = Q(t) + 1? Why or why not? If not, correct the right side of the equation. You may use the functions defined in Section 1.4.
   (b) **(5 pts)** For the same realization considered for the hand simulation in Section 1.4.2, make a plot of L(t) vs. t (similar to Figs. 1.5 and 1.6) between times 0 and T(6).
   (c) **(4 pts)** From your plot in part (b), compute $\hat{l}(6)$ = the time-average number of customers in the system during the time interval [0, T(t)]. Show the expressions for computation, mimicking Equations 1.2 and 1.3.
   (d) **(2 pts)** What is $\hat{l}(6)$ estimating?

Answer:

(a) L(t) = Q(t) + 1 is not true. When the server is idle and the queue is empty, L(t) = 0 < Q(t) + 1 = 1. The right side should be L(t) = Q(t) + B(t), where B(t) is defined on Page 16 in the textbook.

(b)



(c) $\hat{l}(6)$ = ((0.4 – 0) × 0 + (1.6 – 0.4) × 1 + (2.1 – 1.6) × 2 + (2.4 – 2.1) × 3 + (3.1 – 2.4) × 2 + (3.3 – 3.1) × 1 + (3.8 – 3.1) × 0 + (4.0 – 3.8) × 1 + (4.9 – 4.0) × 2 + (5.6 – 4.9) × 1 + (5.8 – 5.6) × 2 + (7.2 – 5.8) × 3 + (8.6 – 7.2) × 4) / 8.6 = 17.6 / 8.6 ≈ 2.05

(d) $\hat{l}(6)$ estimates the expected number of customers in the system.

2. **(20 pts, 4 pts for each event)** Suppose the list of interarrival and service times in Section 1.4.2 is augmented by the following items: A_{10} = 0.6, A_{11} = 1.1, A_{12} = 1.5, A_{13} = 0.8, S_7 = 2.0, S_8 = 2.5. Assume the simulation stops when the 7^{th} customer leaves the system. Show the system state, simulation clock, event list and statistical counters as in Figure 1.7 after processing each even since simulation time of 8.6.

Answer:

Event 1: (Clock = 9.1)

Arrival time = 9.1:
- System: 5.6 (in server), 5.8, 7.2, 9.1 (in queue)

System state:
- Server status = 1
- Number in Queue = 3
- Time of arrival = 5.8, 7.2, 9.1
- Time of last event = 9.1

Event list:
- A = 9.7 (= 9.1 + A_{10} = 9.1 + 0.6), D = 9.2

Statistical counters:
- Number delayed = 6
- Total delay = 5.7
- Area under Q(t) = 10.9 (= 9.9 + (9.1 – 8.6) × 2)
- Area under B(t) = 8.2 (= 7.7 + (9.1 – 8.6) × 1)

Event 2: (Clock = 9.2)

Departure time = 9.2:
- System: 5.8 (in server), 7.2, 9.1 (in queue)

System state:
- Server status = 1
- Number in Queue = 2
- Time of arrival = 7.2, 9.1
- Time of last event = 9.2

Event list:
- A = 9.7, D = 11.2 (= 9.2 + S_7 = 9.2 + 2.0)

Statistical counters:
- Number delayed = 7
- Total delay = 9.1 (= 5.7 + (9.2 – 5.8))
- Area under Q(t) = 11.2 (= 10.9 + (9.2 – 9.1) × 3)
- Area under B(t) = 8.3 (= 8.2 + (9.2 – 9.1) × 1)

Event 3: (Clock = 9.7)

Arrival time = 9.7:
- System: 5.8 (in server), 7.2, 9.1, 9.7 (in queue)

System state:
- Server status = 1
- Number in Queue = 3
- Time of arrival = 7.2, 9.1, 9.7
- Time of last event = 9.7

Event list:
- A = 10.8 (= 9.7 + A_{11} = 9.7 + 1.1), D = 11.2

Statistical counters:
- Number delayed = 7
- Total delay = 9.1
- Area under Q(t) = 12.2 (= 11.2 +(9.7 – 9.2) × 2)
- Area under B(t) = 8.8 (= 8.3 + (9.7 – 9.2) × 1)

Event 4: (Clock = 10.8)

Arrival time = 10.8:
- System: 5.8 (in server), 7.2, 9.1, 9.7, 10.8 (in queue)

Event list:
- A = 12.3 ((= 10.8 + A_{12} = 10.8 + 1.5), D = 11.2

Statistical counters:

System state:
- Server status = 1
- Number in Queue = 4
- Time of arrival = 7.2, 9.1, 9.7, 10.8
- Time of last event = 10.8

Number delayed = 7
Total delay = 9.1
Area under Q(t) = 15.5 (= 12.2 + (10.8 – 9.7) × 3)
Area under B(t) = 9.9 (= 8.8 + (10.8 – 9.7) × 1)

Event 5: (Clock = 11.2) (the 7th customer leaves the system)

Departure time = 11.2:
- System: 7.2 (in server), 9.1, 9.7, 10.8 (in queue)

System state:
- Server status = 1
- Number in Queue = 3
- Time of arrival = 9.1, 9.7, 10.8
- Time of last event = 11.2

Event list:
- A = 12.3, D = 13.7 (= 11.2 + S_8 = 11.2 + 2.5)

Statistical counters:
- Number delayed = 8
- Total delay = 13.1 (= 9.1 + (11.2 – 7.2))
- Area under Q(t) = 17.1 (= 15.5 + (11.2 – 10.8) × 4)
- Area under B(t) = 10.3 (= 9.9 + (11.2 – 10.8) × 1)

3. **(4 pts)** Suppose the simulation in section 1.4 stops at simulation time of 8.1. Show the system state, simulation clock, event list and statistical counters at the end of the simulation.

Answer:

System state:
- Server status = 1
- Number in Queue = 3
- Time of arrival = 5.6, 5.8, 7.2
- Time of last event = 7.2

Clock = 8.1

Event list:
- A = 9.1, D = 8.6

Statistical counters:
- Number delayed = 5
- Total delay = 2.7
- Area under Q(t) = 5.7
- Area under B(t) = 6.3

4. For the single-server queueing system in Section 1.4, suppose a customer leaves the system without being served after staying in the queue for 0.2 or more units of time.
   (a) **(4 pts)** What new events must be introduced?
   (b) **(60 pts, 4 pts for each event)** Show the system state, simulation clock and event list after each event until the end of the simulation. The simulation finishes at simulation time of 9.0.

Answer:

(a) A new event of "Pre-mature departure" (meaning departure without being serviced) must be introduced.

(b)

Event 1: (Clock = 0.4)

Arrival time = 0.4:
- System: 0.4 (in server)

System state:
- Server status = 1
- Number in Queue = 0
- Time of arrival = (None)
- Time of last event = 0.4

Event list: (P: Premature departure)
- A = 1.6, D = 2.4, P = ∞

Statistical counters:
- Number delayed = 1
- Total delay = 0
- Area under Q(t) = 0
- Area under B(t) = 0

Event 2: (Clock = 1.6)

Arrival time = 1.6:
- System: 0.4 (in server), 1.6 (in queue)

System state:
- Server status = 1
- Number in Queue = 1
- Time of arrival = 1.6
- Time of last event = 0.4

Event list:
- A = 2.1, D = 2.4, P = 1.8

Statistical counters:
- Number delayed = 1
- Total delay = 0
- Area under Q(t) = 0
- Area under B(t) = 1.2 (= (1.6 – 0.4) × 1)

Event 3: (Clock = 1.8)

Premature departure time = 1.8:
- System: 0.4 (in server)

System state:
- Server status = 1
- Number in Queue = 0
- Time of arrival = (None)
- Time of last event = 1.8

Event list:
- A = 2.1, D = 2.4, P = ∞

Statistical counters:
- Number delayed = 1
- Total delay = 0
- Area under Q(t) = 0.2 (= (1.8 – 1.6) × 1)
- Area under B(t) = 1.4 (= 1.2 + (1.8 – 1.6) × 1)

Event 4: (Clock = 2.1)

Arrival time = 2.1:
- System: 0.4 (in server), 2.1 (in queue)

System state:
- Server status = 1
- Number in Queue = 1
- Time of arrival = 2.1
- Time of last event = 2.1

Event list:
- A = 3.8, D = 2.4, P = 2.3

Statistical counters:
- Number delayed = 1
- Total delay = 0
- Area under Q(t) = 0.2
- Area under B(t) = 1.7 (= 1.4 + (2.1 - 1.8) × 1)

Event 5: (Clock = 2.3)

Premature departure time = 2.3:
- System: 0.4 (in server)

System state:
- Server status = 1
- Number in Queue = 0
- Time of arrival = (None)
- Time of last event = 2.3

Event list:
- A = 3.8, D = 2.4, P = ∞

Statistical counters:
- Number delayed = 1
- Total delay = 0
- Area under Q(t) = 0.2 + (2.3 – 2.1) × 1 = 0.4
- Area under B(t) = 1.7 + (2.3 – 2.1) × 1 = 1.9

Event 6: (Clock = 2.4)

Departure time = 2.4:
- System: idle

System state:
- Server status = 0
- Number in Queue = 0
- Time of arrival = (None)
- Time of last event = 2.4

Event list:
- A = 3.8, D = ∞, P = ∞

Statistical counters:
- Number delayed = 1
- Total delay = 0
- Area under Q(t) = 0.4
- Area under B(t) = 1.9 + (2.4 – 2.3) × 1 = 2.0

Event 7: (Clock = 3.8)

Arrival time = 3.8:

Event list:

System: 3.8 (in server)

System state:

Server status = 1
Number in Queue = 0
Time of arrival = (None)
Time of last event = 3.8

A = 4.0, D = 4.9, P = ∞

Statistical counters:

Number delayed = 2
Total delay = 0
Area under Q(t) = 0.4
Area under B(t) = 2.0

Event 8: (Clock = 4.0)

Arrival time = 4.0:

System: 3.8 (in server), 4.0 (in queue)

System state:

Server status = 1
Number in Queue = 1
Time of arrival = 4.0
Time of last event = 4.0

Event list:

A = 5.6, D = 4.9, P = 4.2

Statistical counters:

Number delayed = 2
Total delay = 0
Area under Q(t) = 0.4
Area under B(t) = 2.0 + (4.0 – 3.8) × 1 = 2.2

Event 9: (Clock = 4.2)

Premature departure time = 4.2:

System: 3.8 (in server)

System state:

Server status = 1
Number in Queue = 0
Time of arrival = (None)
Time of last event = 4.2

Event list:

A = 5.6, D = 4.9, P = ∞

Statistical counters:

Number delayed = 2
Total delay = 0
Area under Q(t) = 0.4 + (4.2 – 4.0) × 1 = 0.6
Area under B(t) = 2.2 + (4.2 – 4.0) × 1 = 2.4

Event 10: (Clock = 4.9)

Departure time = 4.9:

System: idle

System state:

Server status = 0
Number in Queue = 0
Time of arrival = (None)
Time of last event = 4.9

Event list:

A = 5.6, D = ∞, P = ∞

Statistical counters:

Number delayed = 2
Total delay = 0
Area under Q(t) = 0.6
Area under B(t) = 2.4 + (4.9 - 4.2) × 1 = 3.1

Event 11: (Clock = 5.6)

Arrival time = 5.6:

System: 5.6 (in server)

System state:

Server status = 1
Number in Queue = 0
Time of arrival = (None)
Time of last event = 5.6

Event list:

A = 5.8, D = 9.2, P = ∞

Statistical counters:

Number delayed = 3
Total delay = 0
Area under Q(t) = 0.6
Area under B(t) = 3.1

Event 12: (Clock = 5.8)

Arrival time = 5.8:

System: 5.6 (in server), 5.8 (in queue)

System state:

Event list:

A = 7.2, D = 9.2, P = 6.0

Statistical counters:

Server status = 1
Number in Queue = 1
Time of arrival = 5.8
Time of last event = 5.8

Number delayed = 3
Total delay = 0
Area under Q(t) = 0.6
Area under B(t) = 3.1 + (5.8 – 5.6) × 1 = 3.3

Event 13: (Clock = 6.0)

Premature departure time = 6.0:
System: 5.6 (in server)
System state:
Server status = 1
Number in Queue = 0
Time of arrival = (None)
Time of last event = 6.0

Event list:
A = 7.2, D = 9.2, P = ∞
Statistical counters:
Number delayed = 3
Total delay = 0
Area under Q(t) = 0.6 + (6.0 – 5.8) × 1 = 0.8
Area under B(t) = 3.3 + (6.0 - 5.8) × 1 = 3.5

Event 14: (Clock = 7.2)

Arrival time = 7.2:
System: 5.6 (in server), 7.2 (in queue)
System state:
Server status = 1
Number in Queue = 1
Time of arrival = 7.2
Time of last event = 7.2

Event list:
A = 9.1, D = 9.2, P = 7.4
Statistical counters:
Number delayed = 3
Total delay = 0
Area under Q(t) = 0.8
Area under B(t) = 3.5 + (7.2 – 6.0) × 1 = 4.7

Event 15: (Clock = 7.4)

Premature departure time = 7.4:
System: 5.6 (in server)
System state:
Server status = 1
Number in Queue = 0
Time of arrival = (None)
Time of last event = 7.4

Event list:
A = 9.1, D = 9.2, P = ∞
Statistical counters:
Number delayed = 3
Total delay = 0
Area under Q(t) = 0.8 + (7.4 – 7.2) × 1 = 1.0
Area under B(t) = 4.7 + (7.4 – 7.2) × 1 = 4.9

The following event is optional.

Event 16: (Clock = 9.0)

Simulation end time = 9.0:
System: 5.6 (in server)
System state:
Server status = 1
Number in Queue = 0
Time of arrival = (None)
Time of last event = 9.0

Event list:
A = 9.1, D = 9.2, P = ∞
Statistical counters:
Number delayed = 3
Total delay = 0
Area under Q(t) = 1.0
Area under B(t) = 4.9 + (9.0 – 7.4) × 1 = 6.5