CSCI 410: Modeling and Simulation

Written Assignment 5 Solutions

(See the attached Excel sheet for calculation details)

1. Suppose a simulation model has been built to simulate the processing time of products on a production line. The output of each simulation run consists of 36 processing times. Suppose 10 pilot runs (or replications) have been conducted and the output is provided in the first worksheet of the attached Excel spreadsheet show the simulation output.
   1) **(15 pts)** If we would like to estimate the expected average time for finishing a task with an absolute error of 0.2 and a confidence level of 95 percent, how many more replications of simulations need to be done? The results of trial calculations must be shown.
   2) **(5 pts)** Suppose more simulations have been run and the data are provided in the second worksheet. Use the amount of data according to the result of sub-question 1 to estimate the expected average time for finishing a task and provided a confidence interval at the level of 95 percent.

Answer:

1) $S^2(10) = 0.2312$

   25 is the smallest i value that satisfies $t_{i-1,0.975}\sqrt{\frac{S^2(10)}{i}} \leq 0.2$

   Therefore, 15 more replications of simulations need to be done.

2) The 95% confidence interval is $\bar{X}(25) \pm t_{24,0.975}\sqrt{\frac{s^2(25)}{25}}$ and is (8.9362, 9.2922).

2. **(15 pts)** Redo Problem 1 for a relative error of 0.03, using the formula to calculate the number of simulation replications needed.

Answer:

1) $S^2(10) = 0.2312$
   $\bar{X}(10) = 9.0744$

   16 is the smallest i value that satisfies $\frac{t_{i-1,0.975}\sqrt{\frac{S^2(10)}{i}}}{\bar{X}(10)} \leq \frac{0.03}{1+0.03} \approx 0.02913$

   Therefore, 6 more replications of simulations need to be done.

2) The 95% confidence interval is $\bar{X}(16) \pm t_{15,0.975}\sqrt{\frac{s^2(16)}{16}}$ and is (8.9434, 9.3508).

3. **(15 pts)** Redo Problem 1 for a relative error of 0.03, using the sequential procedure to calculate the number of simulation replications needed.

Answer:

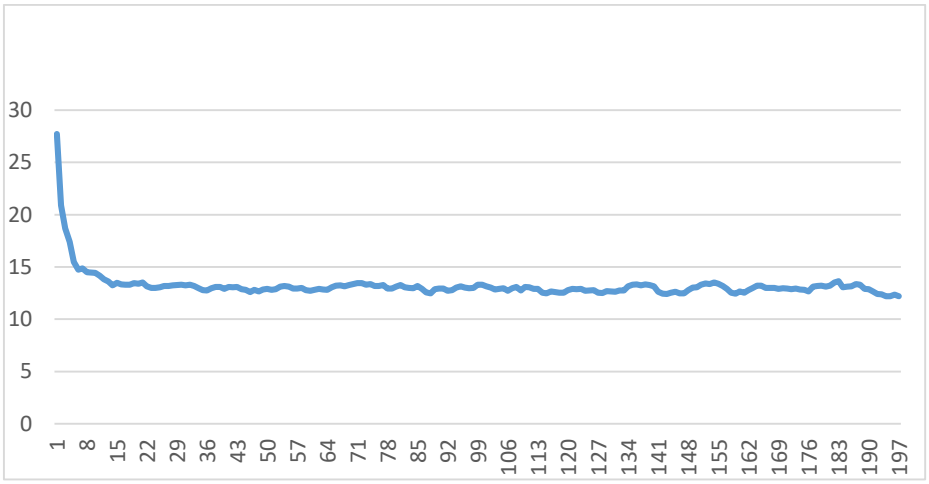
1) 15 is the smallest i value that satisfies $\frac{t_{i-1,0.975}\sqrt{\frac{s^2(i)}{i}}}{\bar{X}(i)} \leq \frac{0.03}{1+0.03} \approx 0.02913$
   Therefore, 5 more replications of simulations need to be done.

2) The 95% confidence interval is $\bar{X}(15) \pm t_{14,0.975}\sqrt{\frac{s^2(15)}{15}}$ and is (8.9484, 9.3798).

4. **(15 pts)** The third worksheet has the output of 100 runs of a simulation. Each number in the output is the time for a computer to execute a certain program. The simulation is known to have the startup problem. Use the Welch's Graphical Procedure and the data of the first 20 runs (or replications) to detect the length of the warmup phase (i.e. the number of output to be disregarded), and then estimate the expected average execution time with a confidence interval at the level of 90 percent. The plots of the tried window size must be shown.

Answer:

Window size 3 is used. The plot is shown below.



As shown in the plot, the smoothened data converges after 20. Therefore, the first 20 output values are discarded in each simulation run and the rest are used for calculate the expected average execution time and the confidence interval. The data from the rest 80 simulation runs are used for the calculations.

The estimated expected average execution time is

$$\bar{X}(80) = 12.86$$

The 90% confidence interval is $\bar{X}(80) \pm t_{79,0.95}\sqrt{\frac{s^2(80)}{80}}$, which is (12.81, 12.91).