SYCS 410
Modeling and Simulation
Fall 2016
Midterm Exam Solutions

(Total points: 100)

(Please PRINT)

Name:

Howard ID:

1. (48 pts) A bank has two cashiers. When a customer comes in, s/he can be served by either available cashier. If both cashiers are busy, a single line is formed, and the customers will be served by the next available cashier in the first-come-first-serve order. The bank would like to know whether they need to add more cashiers. Before making the decision, they need to study customer time spent in the bank, including the waiting time in line and the service time. To perform the study, a simulation is done. The first 10 inter-arrival time (in minutes) of customers are provided as follows. Assume the first customer arrives at time 0.

2.8, 0.2, 9.4, 2.3, 1.2, 18.5, 3.2, 2.4, 2.3, 15.7.

The first 10 service time (in minutes) are provided as follows.

6.3, 0.6, 1.5, 0.8, 4.6, 0.8, 5.4, 4.5, 4.6, 7.4

Show the system state, simulation clock, event list and statistical counters of the simulation for the first 8 events.

$$B(t) = \begin{cases} 2 & \text{if both servers are busy at time t} \\ 1 & \text{if only one server is busy at time t} \\ 0 & \text{if both servers are idle at time t} \end{cases}$$

Arrival time = 0

System state:

- Server 1 status = 1
- Server 2 status = 0
- Number in Queue = 0
- Times of arrival = N/A
- Time of last event = 0

Event list:

- A = 2.8 D = 6.3, ∞

Statistical counters:

- Number delayed = 1
- Total delay = 0
- Area under Q(t) = 0
- Area under B(t) = 0

Arrival time = 2.8

System state:

- Server 1 status = 1
- Server 2 status = 1
- Number in Queue = 0
- Times of arrival = N/A
- Time of last event = 2.8

Event list:

- A = 3.0 D = 6.3, 3.4

Statistical counters:

- Number delayed = 2
- Total delay = 0
- Area under Q(t) = 0
- Area under B(t) = 2.8

Arrival = 3.0

System state:

- Server 1 status = 1
- Server 2 status = 1
- Number in Queue = 1
- Times of arrival = 3.0
- Time of last event = 3.0

Event list:

- A = 12.4 D = 6.3, 3.4

Statistical counters:

- Number delayed = 2
- Total delay = 0
- Area under Q(t) = 0
- Area under B(t) = 3.2

Departure time = 3.4
System state:
Server 1 status = 1
Server 2 status = 1
Number in Queue = 0
Times of arrival = N/A
Time of last event = 3.4

Event list:
A = 12.4 D = 6.3, 4.9
Statistical counters:
Number delayed = 3
Total delay = 0.4
Area under Q(t) = 0.4
Area under B(t) = 4.0

Departure time = 4.9
System state:
Server 1 status = 1
Server 2 status = 0
Number in Queue = 0
Times of arrival = N/A
Time of last event = 4.9

Event list:
A = 12.4 D = 6.3, ∞
Statistical counters:
Number delayed = 3
Total delay = 0.4
Area under Q(t) = 0.4
Area under B(t) = 7.0

Departure time = 6.3
System state:
Server 1 status = 0
Server 2 status = 0
Number in Queue = 0
Times of arrival = N/A
Time of last event = 6.3

Event list:
A = 12.4 D = ∞, ∞
Statistical counters:
Number delayed = 3
Total delay = 0.4
Area under Q(t) = 0.4
Area under B(t) = 8.4

Arrival/departure time = 12.4
System state:
Server 1 status = 1
Server 2 status = 0
Number in Queue = 0
Times of arrival = N/A
Time of last event = 12.4

Event list:
A = 14.7 D = 13.2, ∞
Statistical counters:
Number delayed = 4
Total delay = 0.4
Area under Q(t) = 0.4
Area under B(t) = 8.4

Arrival/departure time = 13.2
System state:
Server 1 status = 0
Server 2 status = 0
Number in Queue = 0
Times of arrival = N/A
Time of last event = 13.2

Event list:
A = 14.7 D = ∞, ∞
Statistical counters:
Number delayed = 4
Total delay = 0.4
Area under Q(t) = 0.4
Area under B(t) = 9.2

2. (42 pts) 30 samples are provided in the following table.
   a) Estimate their mean and variance.
   b) Estimate the skewness. Is the distribution symmetric or not?
   c) Calculate the 90% confidence interval.
   d) Calculate the 90% t confidence interval.
   e) Calculate the 90% Willink confidence interval for the mean.
   f) Test the null hypothesis H_0: μ = 2.1 at the level α = 0.1.

1.94	1.02	-7.03	1.42	3.44	4.91	-0.30	2.34	1.26	-0.05
3.63	7.32	-3.23	4.35	-0.45	-0.24	1.13	3.52	2.01	5.23
1.10	1.78	3.40	5.32	6.21	1.55	-3.14	8.60	1.57	4.84

Answer:

a) $\bar{X}(30) = \frac{\sum_i^{30} X_i}{30} = 2.115$

$$S^2(30) = \frac{\sum_{i=1}^{30}[X_i - \bar{X}(30)]^2}{(30-1)} \approx 10.40$$

b) $\hat{\mu}_3 = \frac{30\sum_{i=1}^{30}[X_i - \bar{X}(30)]^3}{(30-1)(30-2)} \approx -18.91$

Estimated skewness $\hat{v} = \frac{\hat{\mu}_3}{[S^2(n)]^{3/2}} = \frac{-18.91}{10.40^{1.5}} \approx -0.56$

Since the estimated skewness is close to 0, the distribution could be symmetric.

c) The confidence interval is $\left[\bar{X}(30) - z_{0.95}\sqrt{\frac{S^2(30)}{30}}, \bar{X}(30) + z_{0.95}\sqrt{\frac{S^2(30)}{30}}\right]$

$z_{0.95} = 1.645$

$$\bar{X}(30) - z_{0.95}\sqrt{\frac{S^2(30)}{30}} = 2.115 - 1.645 \times \sqrt{\frac{10.40}{30}} \approx 1.15$$

$$\bar{X}(30) + z_{0.95}\sqrt{\frac{S^2(30)}{30}} = 2.115 + 1.645 \times \sqrt{\frac{10.40}{30}} \approx 3.08$$

Therefore, the 90% confidence interval is approximately [1.15, 3.08]

d) The confidence interval is $\left[\bar{X}(30) - t_{29.0.95}\sqrt{\frac{S^2(30)}{30}}, \bar{X}(30) + t_{29.0.95}\sqrt{\frac{S^2(30)}{30}}\right]$.

$t_{29,0.95} = 1.699$

$$\bar{X}(30) - t_{29.0.95}\sqrt{\frac{S^2(30)}{30}} = 2.115 - 1.699 \times \sqrt{\frac{10.40}{30}} \approx 1.11$$

$$\bar{X}(30) + t_{29.0.95}\sqrt{\frac{S^2(30)}{30}} = 2.115 + 1.699 \times \sqrt{\frac{10.40}{30}} \approx 3.11$$

Therefore, the t 90% confidence interval is approximately [1.11, 3.11]

e) The confidence interval is $\left[\bar{X}(30) - G(t_{29.0.95})\sqrt{\frac{S^2(30)}{30}}, \bar{X}(30) + G(t_{29.0.95})\sqrt{\frac{S^2(30)}{30}}\right]$.

$$a = \frac{\hat{v}}{6\sqrt{n}} = \frac{-0.56}{6 \quad \times \sqrt{30}} = -0.017$$

$$G(t_{29.0.95}) = G(1.699) = \frac{[1 + 6 \times (-0.017) \times (1.699 + 0.017)]^{1/3} - 1}{2 \times (-0.017)} \approx 1.83$$

$$G(-t_{29.0.95}) = G(-1.699) = \frac{[1 + 6 \times (-0.017) \times (-1.699 + 0.017)]^{1/3} - 1}{2 \times (-0.017)}$$

$$\approx -1.59$$

$$\bar{X}(30) - G(t_{29.0.95})\sqrt{\frac{S^2(30)}{30}} = 2.115 - 1.83 \times \sqrt{\frac{10.40}{30}} \approx 1.03$$

$$\bar{X}(30) - G(-t_{29.0.95})\sqrt{\frac{S^2(30)}{30}} = 2.115 + 1.59 \times \sqrt{\frac{10.40}{30}} \approx 3.05$$

Therefore, the Willink 90% confidence interval is approximately [1.03, 3.05]

f) $t_{30} = \frac{\bar{X}(30) - \mu_0}{\sqrt{\frac{S^2(30)}{30}}} = \frac{2.115 - 2.1}{\sqrt{\frac{10.40}{30}}} \approx 0.0255 < t_{29,0.95} = 1.699$

Therefore, we cannot reject the null hypothesis at the level α = 0.1.

3. (10 pts) A grocery store analyzed the number of shopping customers' day by day. They found out that ρ_1 (the correlation of lag 1) is -0.8, while the correlations of other lags are within the range of (-0.05, 0.05). What can you infer from this information?

Answer:

ρ_1 is close to -1. It means that two numbers next to each other are negatively correlated. In other words, if one day the number of customers is high, the next day it tends to be low, and vice versa. However, since the correlations of other lags are close to 0, there are no obvious correlations between the numbers with at least two days apart.